

Quasi-subtractive varieties

Tomasz Kowalski

Based on a joint work with Francesco Paoli and Matthew Spinks

Abstract: The theory of subtractive varieties, developed by Agliano and Ursini, can be seen as a successful attempt at capturing the essence of a correspondence between congruences and ideal-like subsets of algebras. However, certain well-known correspondences of a similar kind are not covered by subtractivity. One such is the correspondence between congruences and deductive filters in residuated lattices. We will present a generalisation of subtractivity, which covers this correspondence as well as a number of others not encompassed by subtractivity. The crucial technical notion we work with is "open filter".

We will show:

- (1) that every algebra in a quasi-subtractive variety decomposes as a subdirect product of a subtractive algebra and a "flat" one: typically the "flat" factor is responsible for failures of congruence identities.
- (2) That quasi-subtractive algebras admit a construction, we call "open contraction, generalising kernel contractions in residuated lattices.
- (3) That Goedel translation of S4 to intuitionistic logic and Glivenko translation of intuitionistic logic to classical logic, viewed algebraically, can be seen as cases of open contractions.
- (4) That the lattice of open filters is modular, and admits a reasonable notion of commutator.