State as Expectation Operator on Free MV-algebra of Random Variables

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Abstract. Recent years have witnessed an increasing interest in a probability theory built over algebras of many-valued events. Classical probability theory evaluates yes/no Boolean events, whereas many-valued probability calculus assigns probabilities to many-valued events represented by (equivalence classes of) formulas in Łukasiewicz logic. This model became a framework for studying several deep problems concerning de Finetti theorems for many-valued events, Rényi conditionals, hyperreal-valued states etc.

We make an effort to find an alternative interpretation of many-valued events in terms of classical probability theory. In particular, many-valued events will be identified with random variables and, consequently, the free MV-algebra of finitely-valued Łukasiewicz logic will be seen as a special algebra of random variables. The alternative interpretation will be a model of a simple stochastic experiment based on an imprecise observation of elements in a finite MV-chain. Due to the integral representation theorem, a state of the free algebra becomes the expectation operator. We will mention the issues arising from a generalization of this model toward infinite-valued Łukasiewicz logic.