PROXIMITY RELATIONS ON LATTICE-ORDERED ABELIAN GROUPS

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Based on joint work in progress with G. Bezhanishvili, B. Olberding, and P. Morandi.

Fix a compact Hausdorff space X. The collection $\mathfrak{C}$ of clopen (i.e. both closed and open) subsets of X is a Boolean algebra under set-theoretic operations. The collection $\mathfrak{R}$ of regular open (i.e. equal to the interior of their closure) subsets of X is a Boolean algebra, too, though operations are no longer set-theoretic; and $\mathfrak{R}$, unlike $\mathfrak{C}$, is always complete.

From $\mathfrak{C}$ alone, one can recover X (to within a homeomorphism) if, and only if, X is zero-dimensional, i.e. it is a Stone space. This fact embodies the essence of Stone duality for Boolean algebras. From $\mathfrak{R}$ alone, one cannot recover X. However, it turns out that the pair $(\mathfrak{R}, \prec)$ does determine X up to homeomorphism, where $U \prec V$ is the relation "the closure of U is included in V". This fact embodies the essence of *de Vries duality* for compact Hausdorff spaces. An appropriate, abstract counterpart of the relation $\prec$ on a complete Boolean algebra B is called a *proximity relation*, and the pair $(B, \prec)$ is called a *de Vries algebra*.

I will show how to obtain a de Vries-type duality for compact Hausdorff spaces by endowing certain unital lattice-ordered Abelian groups with a proximity-like, binary relation. The key tool for this construction is a natural correspondence between Boolean algebras and certain unital lattice-ordered Abelian groups. This correspondence is a special case of a well-known, general result of Mundici that was first discovered in connection with the algebraic investigation of Łukasiewicz infinite-valued propositional logic.

Finally, I will discuss the potential import of the result presented in the talk, and what we hope to extend it to, if we are lucky enough.

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