On some class of algebras definable by externally compatible identities of MV-algebras

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We will consider MV-algebras as systems $\langle A, +, \cdot, ^{-}, 0, 1\rangle$, where A is a nonempty set of elements, 0 and 1 are distinct constant elements of A, $+$ and $\cdot$ are binary operations on the elements of A, and $^{-}$ is a unary operation on elements of A.

It is known that the set $\mathbf{Id}(\mathcal{MV})$ of all identities fulfilled in the class $\mathcal{MV}$ determines a variety (i.e., nonempty class of algebras closed under subalgebras, homomorphic images and direct products) $\mathcal{MV}$.

An identity is called externally compatible iff has one of the following forms:

$$\varphi_1 = \varphi_1 \tag{1}$$

$$\varphi_1 + \varphi_2 = \psi_1 + \psi_2 \tag{2}$$

$$\varphi_1 \cdot \varphi_2 = \psi_1 \cdot \psi_2 \tag{3}$$

$$\overline{\varphi_1} = \overline{\psi_1}, \tag{4}$$

where $\varphi_1, \varphi_2, \psi_1, \psi_2$ are term of the type $(2, 2, 1, 0, 0)$.

We will choose from the set $\mathbf{Id}(\mathcal{MV})$ the set $\mathbf{Ex}(\mathcal{MV})$ of all externally compatible identities fulfilled in $\mathcal{MV}$. The corresponding variety to the set $\mathbf{Ex}(\mathcal{MV})$ is bigger with respect to inclusion then $\mathcal{MV}$.

Subvarieties of MV-algebras have been studied by R. Grigolia, Y. Komori, A. Di Nola, and A. Lettieri. A. Lettieri and A. Di Nola gave equational bases for all MV-varieties. Y. Komori determined the lattice of subvarieties of the variety of MV-algebras.

We will ask for the $\mathbf{L}(\mathcal{MV}_{\mathbf{Ex}})$ lattice of subvarieties of the variety defined by the set $\mathbf{Ex}(\mathcal{MV})$. The full description of the lattice $\mathbf{L}(\mathcal{MV}_{\mathbf{Ex}})$ is complicated and falls outside the scope of the talk. Of course, every subvariety of the class $\mathcal{MV}$ is also a proper subvariety of the variety determined the set $\mathbf{Ex}(\mathcal{MV})$.

Beside giving a description of some chosen elements of the lattice $\mathbf{L}(\mathcal{MV}_{\mathbf{Ex}})$ we will give a description of subdirectly irreducible algebras from the classes in the lattice $\mathbf{L}(\mathcal{MV}_{\mathbf{Ex}})$.

References

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