CRAWLEY COMPLETIONS AND SEMANTICS FOR SUBSTRUCTURAL PREDICATE LOGICS

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We consider here semantics for substructural logics. It is known that residuated lattices provide us algebraic semantics for substructural propositional logics. When we extend algebraic semantics to one for substructural *predicate* logics, a standard way is to interpret universal and existential quantifiers as infinite meets and joins, respectively. To guarantee the existence of infinite meets and joins, for our *algebraic frames* we need to take structures of the form $\langle \mathbf{A}, V \rangle$ with a *complete* residuated lattice $\mathbf{A}$ and a nonempty set V for individual domain.

Then the main issue of showing algebraic completeness of a given substructural predicate logic $\mathbf{L}$ is to find a "good" complete algebra. A completion of a given algebra $\mathbf{A}$ is defined to be a pair $(\mathbf{C}, h)$ of a complete algebra $\mathbf{C}$ and an embedding $h : \mathbf{A} \to \mathbf{C}$. For our purpose, we take a Lindenbaum algebra of $\mathbf{L}$ for $\mathbf{A}$. But the interpretation of quantifiers in $\mathbf{A}$ must be also preserved by this embedding. Thus, we need *regular completions*, i.e. completions with *regular* embeddings which preserve all existing joins and meets. MacNeille completions are typical examples of regular completions. By using them, we can show the algebraic completeness of minimum predicate extensions of standard substructural propositional logics. But MacNeille completions do not always preserve distributivity.

In the present talk, we focus our attention on *Crawley completions* (or, complete ideal completions) of residuated lattices, in comparison with MacNeille completions. The Crawley completion of a given residuated lattice $\mathbf{A}$ is an algebra consisting of all complete ideals of $\mathbf{A}$, i.e. such ideals J that for each nonempty subset S of J the join $\bigvee S$ belongs to J if it exists. They are always regular and preserve infinite join distributivity. Using preservation results under Crawley completions, we can show algebraic completeness of minimum predicate extensions of many substructural propositional logics with both the distributivity and the axiom scheme $(\wedge, \exists) :\ \exists x\varphi(x) \wedge \psi \to \exists x(\varphi(x) \wedge \psi)$. We discuss also connections between ideal completions and Kripke-type semantics for substructural propositional logics by Ono-Komori (1985), and between Crawley completions and Kripke-type semantics for substructural predicate logics by Ono (1985). The contents of my talk will be discussed in details in "Regular completions, infinite distributivity and completeness of substructural predicate logics".

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